ARTICLE NO. 79-0156R

Simulation of Liquid Propellant Rocket Engine Combustion Instabilities

M. B. Ventrice*
Tennessee Technological University, Cookeville, Tenn.
J.C. Fang†
Aerojet Liquid Rocket Co., Sacramento, Calif.
K.R. Purdy‡
Tech Consultants, Inc., Smyrna, Ga.

A simulation technique for studying the high-frequency combustion instabilities of liquid propellant rocket engines has been developed and used to investigate various aspects of instability phenomena. Of importance was investigation of the significance of the method of coupling the combustion and the gasdynamics of the system. Two coupling processes were studied: 1) linear response of the combustion process to pressure fluctuations, and the nature of the resulting instabilities; and 2) nonlinear response of the combustion process to velocity fluctuations, and the nature of the resulting instabilities. For the combustion model studied, nonlinear (velocity) coupling was found to characterize liquid propellant instabilities more closely.

Introduction

THE object of the research was to develop an analog system for the study of high-frequency combustion instabilities in liquid propellant rocket combustion chambers and to use the analog to investigate the basic mechanism governing these instabilities. Before proposing an analog, an understanding of the phenomena being modeled was established. During combustion in liquid propellant rocket engines, the combustion process and the gasdynamic processes are coupled. Any oscillation in gasdynamic variables causes an oscillatory burning rate; and the oscillatory burning process serves as the energy source to sustain gasdynamic oscillations. Under certain conditions, when the energy supplied by an unsteady combustion process is sufficient to overcome the energy losses in the gasdynamic field, the coupling is self-sustained and the oscillations can build to large amplitudes, a condition referred to as unstable combustion. So, in part, the questions to be addressed by the analog were why and how the random oscillations that normally occur during operation of a rocket engine change character and develop into an organized pattern of acoustic oscillations which can build in magnitude and very quickly destroy the engine if allowed to continue.

To characterize the combustion process, Priem's combustion model[1] in which the combustion is viewed as vaporization limited, was selected. In this model the burning rate is equal to the vaporization rate of the liquid propellant droplets, and this vaporization or burning rate $\mathcal{W}$ is given as

$$\mathcal{W} = C_l + C_2 Re^{0.5} \quad (1a)$$

where

$$C_l = \frac{2\pi D M_l \mathfrak{D}_d}{R_u T} P\ell n \frac{P}{P-P_v} \quad (1b)$$

$$C_2 = \frac{0.6 S_c{}^{0.33} \pi D M_l \mathfrak{D} C_d}{R_u T} P\ell n \frac{P}{P-P_v} \quad (1c)$$

and

$$Re = \frac{\rho D | V - V_d |}{\mu} \quad (1d)$$

where D is the inside diameter of the combustion chamber, M_l is the molecular weight of the propellant, $\mathfrak{D}$ is the molecular diffusion coefficient, C_d is the concentration of propellant drops, R_u is the universal gas constant, T is the gas temperature, P is the gas pressure, P_v is the vapor pressure of the fuel, S_c is the Schmidt number, ρ is the gas density, V is the gas velocity, V_d is the propellant droplet velocity, and μ is the dynamic viscosity. Equations (1) indicate that the vaporization rate is convection limited (Reynolds number dependent) with additional sensitivity to the gasdynamic and droplet variables.

In Preim's model, the burning rate is sensitive to the magnitude of the surrounding gas velocity with respect to the drop velocity, i.e., it is sensitive to a "rectified" relative velocity. This causes higher harmonic frequency components of the burning rate to occur in response to fundamental frequency perturbations of the gasdynamic field. In addition to this method of generating higher harmonic components of the burning rate, and consequently of the gasdynamic variables, nonlinear gasdynamic effects will also generate higher harmonic components of the gasdynamic variables.

The Analog

The analog selected to model the vaporization-limited combustion process was a hot-wire sensor and the associated series output of a constant-temperature hot-wire anemometer, a phase change or time delay device, a power amplifier, and an acoustic driver. The heart of the analog is the anemometer, which applies a voltage across the hot wire, causing it to heat to some preset temperature (resistance) level. Any flow that passes over the wire tends to cool it, but the anemometer responds to counteract this by increasing the voltage across the wire. The bridge voltage of the anemometer E is related to the flow over the wire. Fang[2] expressed this relationship as

$$E^2 = \frac{[R_w + R_{cbl} + R_p + R_s]^2}{R_w} [C_3 + C_4 Re_f{}^m] \quad (2a)$$

where

$$C_3 = \pi L_w k_f [T_f / T]^{0.17} [T_w - T] A \quad (2b)$$

Presented as Paper 79-0156 at the AIAA 17th Aerospace Sciences Meeting, New Orleans, La., Jan. 15-17, 1979; submitted Feb. 14, 1979; revision received May 18, 1979. This paper is declared a work of the U.S. Government and therefore is in the public domain. Reprints of this article may be ordered from AIAA Special Publications, 1290 Avenue of the Americas, New York, N.Y. 10019. Order by Article No. at top of page. Member price $2.00 each, nonmember, $3.00 each. **Remittance must accompany order.**

Index category: Combustion Stability, Ignition, and Detonation.

*Associate Professor, Dept. of Mechanical Engineering. Member AIAA.

†Engineering Specialist.

‡Principal Research Engineer.

$$C_4 = \pi L_w k_f [T_f/T]^{0.17} [T_w - T] B \tag{2c}$$

and

$$Re_f = \frac{\rho_f D_w |V|}{\mu_f} \tag{2d}$$

where D_w is the diameter of the hot wire; R_w is the operating resistance of the hot wire; R_{cbl} is the resistance of the cable connecting the hot wire and probe to the anemometer; R_p is the probe resistance; R_s is the appropriate anemometer resistance; m, A, and B are coefficients whose exact values are determined by calibration of the individual hot wire (typical values are $m = 0.4442$, $A = 0.5205$, and $B = 0.6729$); L_w is the length of the hot wire; k_f is the gas (air) thermal conductivity evaluated at the film temperature $T_f \equiv [T + T_w]/2$; T is the gas temperature; T_w is the hot-wire operating temperature; ρ_f is the gas density evaluated at the film temperature; V is the component of the gas velocity normal to the stationary hot wire; and μ_f is the dynamic viscosity evaluated at the film temperature. (Other discussions of the development of Eqs. (2) are given in several references by Purdy, Ventrice, and Fang.[3-5] Equations (2) indicate that the square of the anemometer output voltage is analogous to the convection limited burning rate $\mathcal{W}$ given by Eqs. (1). To complete the analogy, a phase change or time delay device is used as the analog of the phase change or time delay associated with combustion; a power amplifier is used as the analog of the energy released during the burning of the propellant; and an acoustic driver is used to convert the energy associated with the anemometer and power amplifier to gasdynamic energy.

In the analog model, the anemometer output is dependent upon the magnitude of the gas velocity normal to the hot wire, i.e., it is sensitive to a "rectified" relative velocity. As in combustion, this causes higher harmonic frequency components of the anemometer output to occur in response to fundamental frequency oscillations in the gas velocity field. In addition to the nonlinearity of the analog process resulting in higher harmonic components of the anemometer output voltage, and consequently of the gasdynamic variables, nonlinear gasdynamic effects also generate higher harmonic perturbations of the gasdynamic variables.

A series of events, analogous to those which occur in the combustion system, occur in the analog system. Acoustic perturbations in the gasdynamic field are sensed by the hot wire. The resulting anemometer output signal is amplified, after phase change or time delay, and fed to the acoustic driver, which generates gasdynamic perturbations proportional to the signal to it. These new acoustic perturbations combine with the previous ones to form a new gasdynamic field. As in combustion, in some situations the perturbations will reinforce each other and be sustained or increased in intensity; in other cases they will die out.

Some comment is necessary as to the details of the location of the components of the analog. Each microphone or hot-wire sensor is responsive to the perturbations in the small region of space surrounding it, and the energy is introduced into the chamber over the area of the acoustic driver outlet. It is appropriate to examine the conditions in these small regions of the chamber and then to interpret the results on a larger scale, since the large-scale results are a combination of local effects.

When positioning the sensors and drivers, the first idea which might come to mind is to place the sensors immediately adjacent to the driver outlet so as to closely couple the sensor and resulting energy release. However, this was not actually found to be the best location as there is some distortion of the acoustic field in the immediate vicinity of the driver outlet. A better relative orientation of sensor and driver outlet was established by looking at the perturbations which develop in an actual engine.

During acoustic instability, the perturbations in the vaporization and energy release of the individual fuel drops combine to drive the organized pattern of an acoustic disturbance. Solutions of the inviscid wave equation[2] yield the pressure and velocity distribution for various modes of acoustic perturbations. In combustion instability the first tangential (1T) resonant mode is dominant at some fundamental frequency f, with a less well-defined acoustic mode occurring at twice the fundamental frequency $2f$. In the transverse plane of a 1T mode, the pressures and velocities at two differemt spots, both having the same value of r but located 180 deg apart, have the same amplitudes but are 180 deg out of phase. Driving such a mode can be done equally well at either of the two locations described, provided that the perturbations in the energy release at the second location is 180 deg out of phase with the energy release at the first. In other words, that particular oscillation could be driven equally well by energy release at the 0-deg location or at the 180-deg location, provided the driving occurring at the 180-deg location was 180 deg out of phase with the driving at 0. Hence, it was decided that in order to avoid the difficulties associated with local irregularities in the acoustic field near the driver outlet location, to drive at the location 180 deg away with the necessary phase change incorporated into the loop.

A comment also needs to be made concerning time delay. Time delay is incorporated into analytical models of the combustion process and can be interpreted as a "history" effect. Any oscillations which occur in the chamber die out or are sustained because of the relative magnitudes of viscous damping and the extent of destructive or constructive interference with new oscillations being generated by instabilities in the combustion process. Oscillations in pressure and velocity caused by unsteady vaporization and burning of a fuel drop affect drops of fuel which are introduced into the chamber at a later time. There are limitations on the length of time during which the effect of the burning of one fuel drop can affect a drop appearing later. The time delay term in analytical models accounts for this factor—it accounts for past history. Time delay is known to be significant in the analytical modeling of the pressure sensitive situation.[6]

A similar factor can be incorporated into the analog device by placing a time delay device in the loop. With this feature, studies can be done on the effect of time delay so as to determine whether or not efforts should be made in an actual system to alter the effective time delay.

Open-Loop Studies

Of most importance in both combustion instability and instability of the analog is how the existing and feedback acoustic pressure fields combine. One of the better ways of relating the two fields is to find the correlation between them. An explicit expression for the feedback acoustic pressure in terms of the burning rate has not been available; hence, for the combustion process, Heidmann[7-9] analytically correlated the existing acoustic pressures with the burning rate itself, instead of the feedback pressure that is produced by the unsteady burning rate. He used harmonic response factors to indicate the degree of reinforcement of the acoustic pressure by the combustion process and found that the response factors depended upon the amplitudes of the harmonic components and the phase between the acoustic pressure and burning rate harmonic components.

The idea for using an anemometer as an analog to combustion was originated by Hribar[10] when he noticed the similarity of the open-loop responses of a liquid fuel droplet's vaporization and a constant-temperature anemometer's energy transfer rate to the same gasdynamic environment. Subsequently Purdy et al.[3] performed an analytical investigation and Ventrice and Purdy[4] carried out an experimental study of the open-loop response of a constant-temperature hot-wire anemometer to special sound fields. The analytical results were found to be similar to Heidman's open-

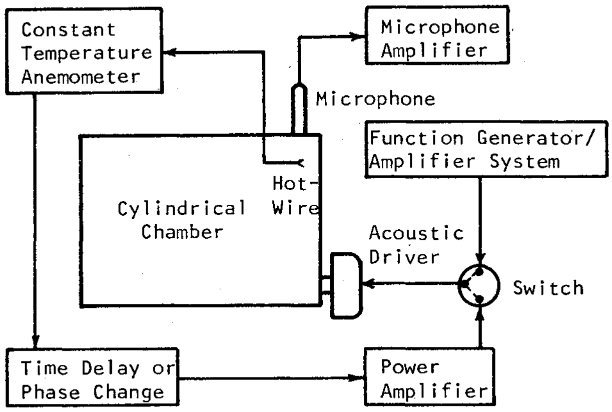

Fig. 1 Schematic diagram of the experimental system.

loop, analytical results for a vaporization-limited combustion process. The experimental results qualitatively agreed with the analytical results; however, there were still some points that remained to be studied before a quantitative comparison could be achieved.

Closed-Loop Experiments

A number of factors were important in selecting the geometry of the chamber used. An actual rocket engine is complex acoustically, since one end consists of an injector plate containing many holes and the other end consists of a nozzle. In the developmental stages of the analog it was not necessary to have an exact duplicate of a rocket engine—it was only necessary to duplicate the essential characteristics. These were those associated with resonance. The resonant characteristics are determined by the chamber shape and dimensions. A cylindrical shape was required in order to be able to generate the appropriate 1T type oscillation. The dimensions are important since they determine the chamber's resonant frequencies. The chamber is more "responsive" to excitation at these frequencies, i.e., an input signal of a fixed level will produce a much more intense sound field if it is at a resonant frequency than if it is at a nonresonant frequency. Also, even for resonant frequencies, the chamber is more responsive to some frequencies than to others. An open-open, open-closed, or closed-closed configuration could establish the significance of resonance. The characteristics of the acoustic oscillations of an actual rocket engine seem to be most closely characterized as those associated with a closed-closed situation.[6] Also, much of the analytical work in the literature has been done with a closed-closed configuration. There were also two practical reasons for using a closed-closed configuration. First, with an open-open or open-closed configuration, much of the energy introduced into the chamber by the acoustic driver is dissipated in the environment; hence, much more powerful drivers and larger capacity power amplifiers would be needed. Second, the sound level in the surroundings for an open-open or open-closed configuration would be much too intense to be tolerated by humans. Even with a closed-closed configuration, which has about a 20-dB drop in sound level between the inside and outside of the chamber, the sould level outside was intense enough to require the test chamber to be surrounded by an acoustic barrier. Hence, considering all aspects, a closed-closed cylindrical chamber was used.

It should be pointed out that the damping caused by a rocket nozzle has no counterpart in the analog system (nor, because of the absence of throughflow, would it in a closed-open chamber). Thus, it is likely that some sets of conditions which would lead to stability in a rocket motor would lead to instability in the analog system. Since the purpose of this research was to study instability, this is thought to be an advantage. Further damping could always be introduced into the system should it become desirable to do so.

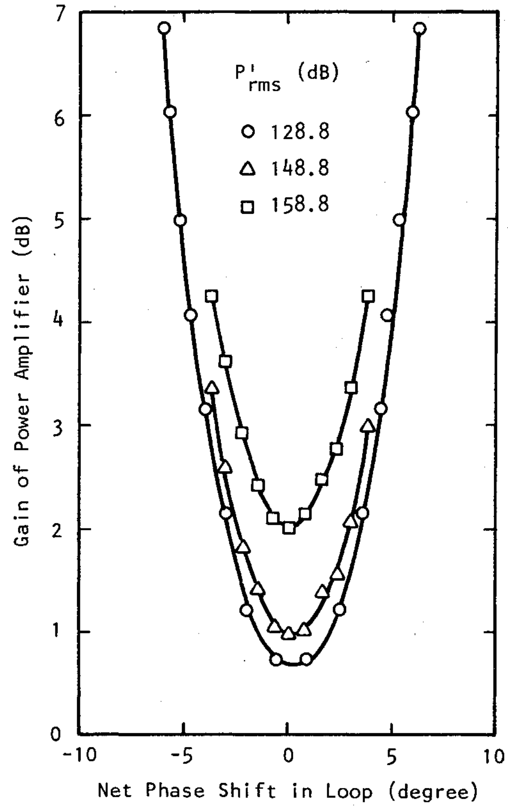

Fig. 2 Feedback-loop gain required to produce given sound-pressure levels vs phase shift—resultant closed-loop sound field frequency $=f_{1T}=992.5$ Hz.

Figure 1 is a schematic diagram of the experimental system. The test section's length-to-diameter ratio could be adjusted to two different values— one in which twice the frequency of the first tangential resonant mode of vibration was equal to the frequency of the second tangential-second longitudinal resonant mode of vibration, $2f_{1T}=f_{2T,2L}$; and one in which twice the frequency of the first tangential was not equal to a resonant frequency of the chamber, $2f_{1T}=f_x$. Two chambers were used to establish the effect of chamber dimensions on instability.

Referring to Fig. 1, the gas-filled chamber could be excited by moving the switch into the upper position so that the function generator/amplifier system supplied the excitation signal to the driver (open-loop operation). By switching to its lower position, closed-loop operation could be obtained. For linear feedback operation, the microphone, instead of the hot wire, was the sensor of disturbances in the chamber, and the resulting microphone output was processed by the microphone amplifier and then went to the time delay unit and through the remainder of the loop.

A comment needs to be made concerning the electrical output signal of the anemometer and its relationship to the anemometer bridge voltage. The bridge voltage is the sum of a dc and an ac component, $E=\overline{E}+E'$; hence $E^2=\overline{E}^2+2\overline{E}E'+(E')^2=(\overline{E^2})+(E^2)'$. The acoustic driver responds only to an ac signal; therefore, only the ac component of the squared voltage is needed as a feedback signal source. When E' is small relative to $\overline{E}$, Fang[2] showed that E' approximates $(E^2)'$ to a satisfactory degree with respect to amplitudes, frequencies, and phases. Hence, E' was used as the feedback signal in the loop.

Linear Feedback Experiments

The purpose of the linear feedback experiments using the microphone as a sensor was to investigate the characteristics of closed-loop system behavior at a single resonant frequency. The general procedure followed was to excite the chamber in the open-loop mode with some predetermined signal by means

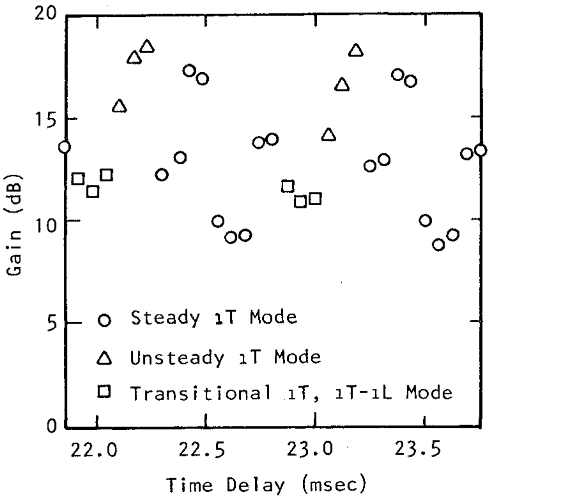

Fig. 3 Self-sustained map of the hot-wire closed-loop system ($2f_{1T} = f_{2T,1L}$ chamber).

of the function generator/amplifier system. The switch was then moved to its alternate (closed-loop) operating position and the system was observed to see what would evolve. Three basic responses could develop: the acoustic pressure in the chamber could die out; the acoustic pressure could build in intensity to such a level that the switch had to be moved back to its open-loop position to avoid burning out the fuse in the driver circuit; or, finally, some type of steady-state acoustic pressure field could be attained. It was the conditions under which the third response occurred that were of interest. This represented a neutral stability situation. During the course of the experiments, several parameters were altered systematically to determine their effect on the closed-loop operation of the system. These were the open-loop signal from the function generator/amplifier system to the driver, the amount of time delay or amount of phase shift, the amount of amplification or gain, and the length-to-diameter ratio of the chamber.

Typically, after closing the loop, the initial open-loop acoustic pressure field died out and a new sound field developed which attained some steady-state sound-pressure level. Figure 2 shows the results. The gain required to produce a given sound-pressure level is plotted as a function of the phase shift for a first tangential (1T) resonant sound field. The three constant sound-pressure-level curves are "parabolic" in form and are symmetric with respect to the line of zero degrees of net phase shift through the loop, i.e., the three optimal phase shifts are the same. The figure indicates that: 1) for a given phase shift, the higher the loop gain, the higher the closed-loop sound-pressure level; and 2) for a given gain, the greater the deviation from the optimal phase shift, the lower the closed-loop sound-pressure level. The "outer" curve ($L_P = 128.8$ dB) is considered to be the minimum gain for which the system is self-sustained for each phase shift. For a given phase shift, and a gain that is lower than the value given by this curve, the system will not produce a self-sustained sound field. Therefore, this curve can be regarded as the lower limit for self-sustenance of the 1T sound field. Ths area contained by this curve is the self-sustenance region whereas that area outside this curve is the nonself-sustenance region. Other similar type data were also obtained.[2]

The overall results of the linear feedback study were as follows.

1) The closed-loop sound field was independent of the character of the initial exciting sound field. The same closed-loop sound field resulted for a particular amount of gain and time delay or phase change in the loop, no matter what the characteristics of the initial open-loop sound field.

Table 1 Net phase change through the loop at various time delays for f_{1T} and $2f_{1T}$

	Net phase change, deg	
Time delay,ms	f_{1T}	$2f_{1T}$
22.69	359	359
22.80	39	78
23.00	111	222
23.21	183	7
23.34	232	104
23.70	360	359

2) For a particular sound field to be self-sustained in closed-loop operation, the least amplification was required when the existing and feedback sound fields were in phase with each other. If these two fields were not in phase with each other, more than this minimum feedback-loop amplification was required in order to have self-sustenance—the greater the phase difference between them, the greater the required feedback gain.

3) In general, the higher the gain, the higher the level of the closed-loop sound field. If self-sustenance did not occur at a certain phase difference between the existing and feedback field, increasing the gain or adjusting the phase difference so that the overall phase change through the loop was closer to a net zero would make the system self-sustaining.

4) Coexistence of two or more frequencies was possible whenever the gain was sufficient for each of them to be self-sustained. They were not normally integral multiples of each other.

5) None but resonant frequencies were self-sustaining.

6) The length-to-diameter ratio of the chamber did not effect the character of the results.

Nonlinear Feedback Experiments

The purpose of the nonlinear feedback experiments was to identify the most important characteristics of self-sustained closed-loop operation of a nonlinear system. The general procedure followed was similar to that of the linear experiments—the chamber was excited in the open-loop mode with some predetermined signal by means of the function generator/amplifier system (see Fig. 1). The switch was then moved to its closed-loop position and the system was observed to determine what type of sound field would evolve. The same three kinds of response could develop: the sound field could die out; the sound field could continuously build in intensity; or it could reach some steady-state operating level. The same parameters could be altered—the open-loop excitation signal, the time delay or phase shift, the gain, and the length-to-diameter ratio of the chamber.

The hot wire itself was contained in the transverse plane located a distance of $5.53 \times 10^{-3} L$ from the end plate at $r = 0.907$R and $\theta = 0$ deg. The wire was turned (but still remained in the transverse plane) so that it formed a 45 deg angle with the radius. In that position it was sensitive to velocity perturbations on the r, θ and z directions.

It should be recalled here that the anemometer's response is nonlinear—it is related to a "rectified" relative velocity. Because of this, its ouput tends to be dominated by a component at $2f$ when exposed to a velocity field of frequency f, i.e., if the chamber was initially excited with an 1T sound field of frequency f_{1T}, the anemometer would have as its output a signal with a strong $2f_{1T}$ component.

It was found that three types of self-sustained operation could occur in the chamber for which $2f_{1T} = f_{2T,2L}$, depending on the time delay or phase change in the loop: 1) steady in waveform and level, having frequency components at f_{1T} and at its higher harmonics; 2) unsteady in waveform and level, having frequency components at f_{1T} and its higher harmonics; and 3) transitional in waveform and level having frequency components at f_{1T}, $f_{1T,1L}$ and higher harmonics of each.

Figure 3 is a map of the lower limit of gain for self-sustenance at various values of time delay, with the type of sound field resulting indicated by the symbols. These results can be partially explained using the results of the linear-feedback experiments and calculations of the net phase change through the loop for the fundamental and second harmonic frequencies, the results of which are listed in Table 1.

First, Fig. 3 follows a repetitive pattern. The basic pattern is that recorded between 22.69 and 23.70 ms. This span of time delay, 1.01 ms, is equal to one period of a signal at f_{1T}. Starting at 22.69 ms, the net phase changes for the f_{1T} and $2f_{1T}$ components are both 359 deg. Since the existing and feedback sound fields are almost in phase, a minimum amount of gain is needed in the loop. At 22.80 ms, where the net phase changes are 39 and 78 deg, respectively, more gain is needed for self-sustenance. At 23.21 ms, the net phase changes are 183 and 7 deg, respectively. Since the fundamental feedback component is almost 280 deg out of phase with the existing fundamental component, the gain required becomes a maximum. When the existing and feedback sound fields are 180 deg out of phase, the level of the feedback sound field must be greater than that of the existing field if self-sustenance is to be attained—the sound fields at f_{1T} are constantly interfering with each other. The level of the self-sustained sound field is unsteady but the frequency components remain the same. This behavior resulted in the closed-loop sound field appearing "unsteady." At a time delay of 23.34 ms—another relative minimum region—the net phase change for the f_{1T} and $2f_{1T}$ components are 232 and 104 deg, respectively. Both values of time delay are significantly different than 180 deg, so less gain is needed for self-sustenance, but not as little gain as at 23.70 ms, where the phase changes have returned to 360 and 359 deg, respectively.

One time delay in Table 1 has not been mentioned—23.00 ms, at which the net phase changes are 111 and 222 deg, respectively. These values of net phase change differ from zero by more than they do at 22.80 ms and, hence, should require more gain instead of less for self-sustenance. However, another factor effects the results at 22.80 ms. As mentioned earlier, a 1T,1L sound field and its higher harmonics sound fields existed along with the 1T sound field and its higher harmonic sound fields in this transitional region. This second series of sound fields reduces the gain needed for self-susteneance. This new sound field appeared in some cases due to the following characteristics: 1) $f_{1T,1L}$ was 1135 Hz while f_{3T} was 2262 Hz. Hence, f_{3T} was approximately twice $f_{1T,1L}$. This is similar to $2f_{1T} = f_{2T,2L}$. It was concluded that whenever a resonant frequency was twice another resonant frequency, a self-sustained closed-loop sound field would be possible if the gain was high enough and the time delay was close enough to an optimal value. For this reason, the appearance of $f_{1T,1L}$ and its higher harmonics, at certain time delays, was to be expected. 2) The 1T,1L sound field requires more energy to drive (it is less responsive) than a 1T sound field. Also, f_{3T} is not exactly twice $f_{1T,1L}$. Hence, for most time delays, the self-sustained sound field is naturally dominated by f_{1T} and its higher harmonics and $f_{1T,1L}$ does not develop.

Another significant finding of the nonlinear experiments was that the resultant closed-loop sound fields do not depend on the form (frequencies and phases) of the exciting sound field. However, the gain required to initiate self-sustenance did vary with the waveform and level of the exciting sound field. When the open-loop sound-pressure levels for various resonant modes were the same, the minimum gain required to initiate self-sustenance occurred when the exciting sound field was the 1T field. Also, for a given exciting sound field, the higher the sound-pressure level, the lower the gain required to initiate self-sustenance.

Other closed-loop nonlinear feedback experiments were also carried out with the chamber for which $2f_{1T} = f_{2T,2L}$. They confirmed the results discussed here. Finally, of great significance, it was not possible to achieve self-sustenance in the chamber for which $2f_{1T}$ was not a resonant frequency.

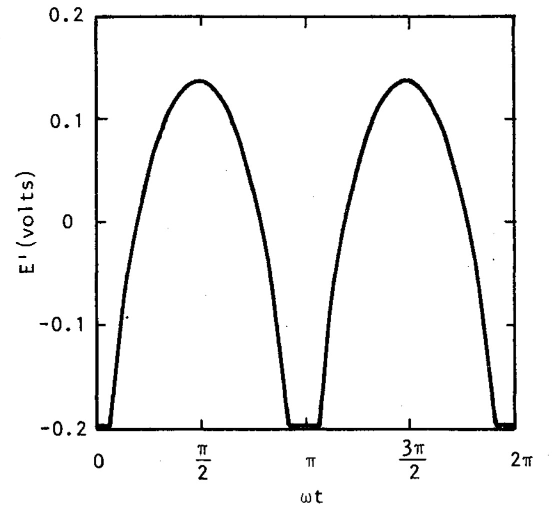

Fig. 4 Analytically predicted ac bridge voltage of the anemometer responding to a 1T sound field.

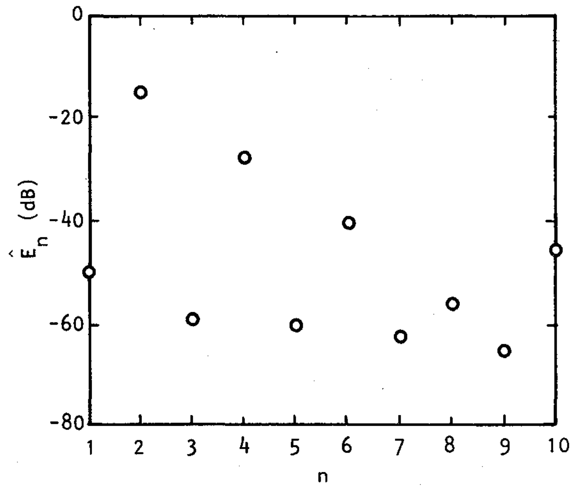

Fig. 5 Harmonic components n of the analytically predicted ac bridge voltage of the anemometer in response to a 1T sound field.

Theoretical Analysis of the Anemometer Output Signal

To understand fully the nonlinear feedback experimental results, it was necessary to analyze the anemometer output which resulted upon exposure of the hot-wire sensor to various sound fields. Considering only the 1T mode, the bridge voltage of the anemometer was calculated. In Fig. 4 the ac bridge voltage is plotted vs dimensionless time. (The curve is flat in the vicinity of $\omega t = 0$, π, and 2π because of the assumption that whenever the calculated values of E, using Eqs. (1), are less than E_0, the no-flow anemometer output E is set equal to E_0. This means that during periods of low or zero air velocity, the bridge voltage is equal to E_0.) Performing a Fourier series analysis, the harmonic components of the analytically predicted ac bridge voltage resulting from a 1T sound field can be calculated. They are shown in Fig. 5. The ac signal is clearly dominated by the second harmonic, which is typical of the rectification process. The presence of the first harmonic is primarily due to the temperature difference terms $(T_w - T)$ in Eqs. (1).

Considering a "distorted" sound field produced by the summation of a 1T and a 2T,2L sound field, the bridge voltage for various values of P_{21} was calculated, where P_{21} is the ratio of the amplitude of the sound pressure of the 2T,2L

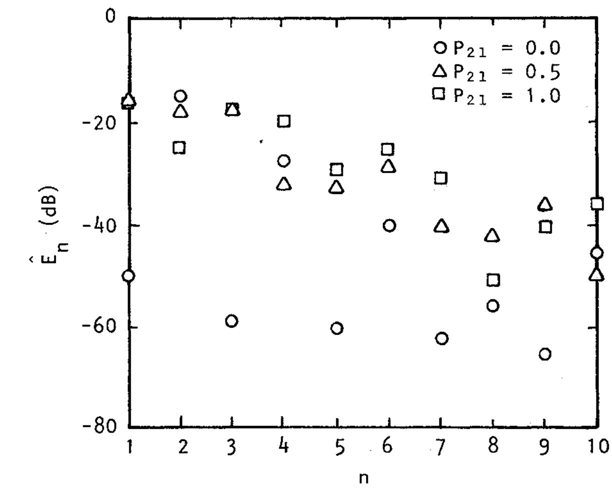

Fig. 6 Harmonic components n of the analytically predicted ac bridge voltage of the anemometer for $P_{2l} = 0$, 0.5, and 1.0.

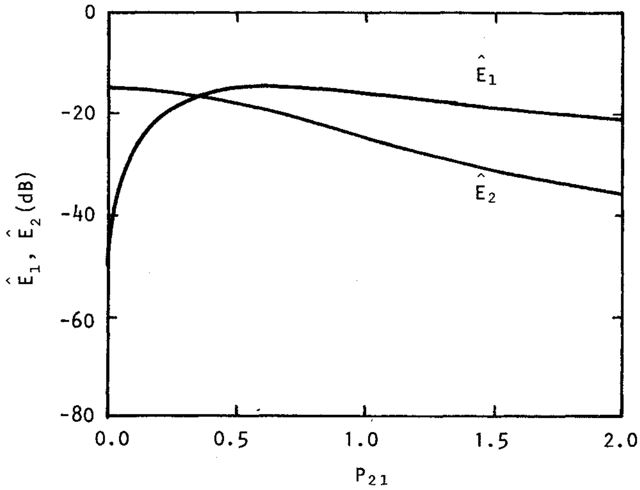

Fig. 7 Variations of the first two harmonic components of the ac anemometer voltage with P_{2l}.

sound field to that of the 1T sound field, and a Fourier series analysis was performed. The results are shown in Figs. 6 and 7. It can be seen that distortion increases the component of E' at the fundamental frequency and that it decreases the component of E' at the second harmonic.

Discussion

From the experimental and analytical nonlinear feedback investigations discussed in the previous two sections, a general description of the transition process from an open-loop sound field to the resultant closed-loop sound field can be given phenomenalogically without describing the details for individual circumstances. Consider that an open-loop sound field at some resonant frequency f is triggered in the chamber. An ac anemometer voltage results which has the frequency characteristics shown in Fig. 5. There are components at all frequencies, f, $2f$, $3f$, etc., but the components at the even frequencies dominate, with the $2f$ component strongest. When this signal continues around the loop and replaces the initial open-loop signal (i.e., when the loop is closed), the produced sound field (feedback sound field) is predominantly at $2f$—twice of the pre-existing sound field.

The acoustics of the chamber are important here. If $2f$ is a resonant frequency of the chamber, a relatively strong $2f$ acoustic field will develop. If it is not, even if the driving at this frequency is strong, a significant acoustic field at $2f$ will not develop. Also, the higher harmonic components of the feedback signal will not result in strong acoustic disturbances at these frequencies, first, because it is unlikely that they are also resonant frequencies, and, second, because even if they were resonant, it is relatively more difficult to drive the higher resonant frequencies that the lower resonant frequencies.

When the two sound fields, the initial at f and the feedback at $2f$, combine, the resulting sound field contains both frequencies. The hot wire senses this new sound field and responds with an ac anemometer voltage that has both fundamental and second harmonic components. Again, the acoustics of the chamber play an important role. The chamber selectively responds at f and $2f$, when these are resonant frequencies, and does not repond well to the higher harmonic components of the input signal. The dependence of the components of the anemometer signal at f and $2f$ on the relative magnitude of the acoustic field in the chamber at f and $2f$ was shown in Fig. 7. Figure 7 indicates that adding even a relatively low-level acoustic field at $2f$ compared to the initial f acoustic field results in a significance component of the feedback signal being at the fundamental frequency f.

Again, the new feedback sound field is superimposed upon the existing sound field, which also consists of the fundamental and second harmonic components, and a new feedback signal is sent to the driver. Similar existing-feedback interaction (closed-loop) processes will continue as long as the sound field exists.

The preceding discussion explains why an acoustic field having components at both f and $2f$ results from the anemometer analog system. The two-frequency characteristic is a result of the nonlinear nature of the feedback mechanism and the acoustics of the chamber. In addition to the factors mentioned, the amount of time delay or phase change in the loop is also of importance. The more out of phase the feedback signal is with respect to the initial perturbation in the chamber, the more amplification will be needed for self-sustenance as was illustrated by Fig. 3.

Because of the nonlinear character of the hot-wire response, all of the self-sustained closed-loop sound fields of the hot-wire closed-loop operation must contain higher harmonic components. If the chamber is not responsive at the $2f$ frequency, i.e., if $2f$ is not a resonant frequency (or close to a resonant frequency), the sound field will not be sustained; hence, the geometry of the chamber is of primary importance.

Conclusions

The hot-wire closed-loop system used in this study was devised to be analogous to a simplified model of the vaporization-limited combustion process in liquid propellant rocket engines; hence, the behavior of this simulation system is expected to be representative of that which would be found in a liquid propellant rocket combustion chamber. The self-sustained situation in the former system corresponds to the acoustic mode unstable combustion condition in the latter system, while a nonself-sustained situation corresponds to the stable combustion condition.

No attempt was made during this work to quantitatively relate the amplification in the analog loop to the amount of amplification in an actual combustion situation. Much more study of both the combustion process and the analog system would have to be carried out to make meaningful comparison. (Work in this area is in progress.) The primary objectives were to first establish the feasibility of setting up such an analog technique and then to use it to gain insight into the phenomena of significance to instability.

The following qualitative comparison of the results of this study and the characteristics of combustion instability may corroborate the similarity between the two systems.

1) In combustion instability, a minimum amplitude of gas disturbance is required to excite instability. In the hot-wire closed-loop system, for a constant time delay, a certain minimum sound-pressure level of the exciting sound field was required.

2) In combustion instability, an initial sinusoidal gas

oscillation becomes harmonically distorted once the instability develops. In the hot-wire system, even with a sinusoidal exciting field, the self-sustained sound field always contains higher harmonic components.

These results indicate that the nonlinear aspect of the combustion process is the dominant mechanism of combustion instability. Nonlinear gasdynamic effects did not appear to affect the process.

This is a simple, relatively inexpensive experimental technique for studying the details of combustion instability and for determining the phenomenalogical aspects of greatest importance. It is much cheaper to use the analog than to carry out actual combustion studies. It has the advantage that parameters can be varied independently so as to see their effect—this cannot be done effectively in an actual combustion situation. Hence, more general studies can be carried out with the analog than are possible in an actual combustion situation. Also, it can be used to study situations too complex for an analytical approach.

What has been reported are the results of developmental studies for an analog system. After some additional work on clarifying further details, it is anticipated that the analog could be applied to specific rocket engines to predict the stability characteristics of those engines.

Acknowledgments

This work was partially supported by NASA Lewis Research Center, Cleveland, Ohio, through research grant NGR 43-003-015, R.J. Priem, Project Monitor. His support is gratefully acknowledged and appreciated. Critical support was also provided by Tennessee Technological University, Department of Mechanical Engineering.

References

[1] Priem, R. J. and Heidmann, M. F., "Propellant Vaporization as a Design Criterion for Rocket-Engine Combustion Chambers," NASA TR R-67, 1960.

[2] Fang, J. C., "On the Convection Limited Self-Sustained Acoustic Vibrations in a Closed-Closed-Cylindrical Chamber," Ph.D. dissertation, Tennessee Technological University, 1975.

[3] Purdy, K. R., Ventrice, M. B., and Fang, J. C., "An Investigation of the Open-Loop Amplification of Reynolds Number Dependent Processes by Wave Distortion," *Proceedings of the Tenth Southeastern Seminar on Thermal Sciences*, April 1974, pp. 176-204.

[4] Ventrice, M. B. and Purdy, K. R., "An Investigation of the Open-Loop Amplification of a Reynolds Number Dependent Process by Wave Distortion," NASA CR-134620, May 1974.

[5] Ventrice, M. B., Fang, J. C., and Purdy, K. R., "Amplification of Reynolds Number Dependent Processes by Wave Distortion," NASA CR-134917, June 1975.

[6] Powell, E. A., "Nonlinear Combustion Instability in Liquid Propellant Rocket Engines," Ph.D. dissertation, Georgia Institute of Technology, 1970.

[7] Heidmann, M. F., "Amplification by Wave Distortion in Unstable Combustors," *AIAA Journal*, Vol. 9, Feb. 1971, pp. 336-339.

[8] Heidmann, M. F., "Amplification by Wave Distortion of the Dynamic Response of Vaporization Limited Combustion," NASA TN D-6287, May 1971.

[9] Heidmann, M. F., "Frequency Response of a Vaporization Process to Distorted Acoustic Disturbances," NASA TN D-6806, May 1972.

[10] Hribar, A. E., "An Investigation of the Open-Loop Amplification of Reynolds Number Dependent Processes by Wave Distortion," Proposal for Research submitted to NASA, June 1971.

Announcement: 1979 Author and Subject Index

The indexes of the six AIAA achive journals (*AIAA Journal, Journal of Aircraft, Journal of Energy, Journal of Guidance and Control, Journal of Hydronautics, Journal of Spacecraft and Rockets*) will be combined and mailed separately early in 1980. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1979 will be included. Librarians will receive one copy of the index for each subscription which they have. Any AIAA member who subscribes to one or more Journals will receive one index. Additional copies may be purchased by anyone, at $10 per copy, from AIAA EDP, Room 730, 1290 Avenue of the Americas, New York, New York 10019. **Remittance must accompany the order.**

Ruth F. Bryans
Administrator, Scientific Publications